Exam Calculus 1

November 2, 2018: 9:00-12:00.

This exam has 8 problems. Each problem is worth 1 point. Total: $8+1$ (free) $=9$ points; more details can be found below. Write on each page your name and student number. The use of annotations, books and calculators is not permitted in this examination. All answers must be supported by work. Success.

1. (a) Formulate the principle of mathematical induction.
   (b) Prove that
   $$1\cdot 3+2\cdot 4+3\cdot 5+\cdots+n(n+2)=\frac{n(n+1)(2n+7)}{6}$$
   for every natural number $n>1$.

2. Find all complex solutions z satisfying
   $$\mathrm{e}^{1/z}=\left(\frac{1+i}{1-i}\right)^2$$

3. (a) The function f is defined on some open interval that contains the number a, except possibly at a itself. Give the precise definition of
   $$\lim_{x\to a} f(x) = L$$
   (b) Prove, using this definition, that
   $$\lim_{x\to 1} x^2-x = 0$$

4. (a) Formulate the Mean Value Theorem.
   (b) Use the Mean Value Theorem to prove the inequality
   $$|\sin a-\sin b| \le |a-b|$$
   for all a and b.

5. Evaluate

(a)

$$\lim_{x \to -\infty} \left(\sqrt{x^2 + 2x} - \sqrt{x^2 - 2x} \right)$$

(b)

$$\lim_{x \to \infty} \left(1 + \frac{2}{x} \right)^x$$

6. (a) Show that the values of the following expression do not depend on x:

$$\int_0^x \frac{1}{1+t^2}\, d\overset{t}{\cancel{x}} + \int_0^{1/x} \frac{1}{1+t^2}\, d\overset{t}{\cancel{x}}$$

(b) Determine $f'(1)$, if

$$f(x) = \int_{x^2}^{0} e^{-\sqrt{t}}\, dt$$

7. Evaluate

(a)

$$\int \ln \sqrt{x}\, dx$$

(b)

$$\int_e^{e^2} \frac{dt}{t \ln t}$$

8. Solve the initial value problem

$$xy'(x) + xy = 1 \qquad x > 0 \qquad y(1) = 2$$

$$\int \frac{e^x}{x}\, dx$$

Maximum points:

1a	0.4	2	1.0	3a	0.4	4a	0.4	5a	0.5	6a	0.5	7a	0.5	8	1.0
b	0.6			b	0.6	b	0.6	b	0.5	b	0.5	b	0.5		